Farmers credit risk modelling under climate uncertainty

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2. Credit Risk
   - Accounting basics
   - Merton’s Model
   - Farm’s Asset Modelling

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   - Two Types of Gaussian Process
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Commodities and Weather Risk

Winter-Spring Mean Rainfall deciles for 12 moderate-strong classical El Niño events

Figure: El Niño background, source: nab
Impact of El Niño

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⇒ who fosters the development of insurance products such as weather, crop or revenue insurance policies which could help the agricultural businesses to overcome the more frequent and damaging weather events.
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In our case we are going to model the dynamic of the assets ↔ dynamic of the liabilities → dynamic of the profits.

The profits are thus partially explaining the dynamic of the assets.
Merton’s Model

Merton’s Main Assumptions

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The term structure is flat and known with certainty which means that at time $t$ a $1$ nominal bond value of maturity $T$ equals:

$$P(t, T) = e^{-r(T-t)}$$

Where $r$ is the risk free rate.

We can describe the value of the firm, $V$, with a diffusion type stochastic process:

$$dV_t = V_t \left( \left[ r - \delta \right] dt + \sigma dW_t \right)$$

With $V_0 > 0$ and $\delta$ is the constant cash-flow payout ratio.
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- We can describe the value of the firm, $V$, with a diffusion type stochastic process.

- Value of the assets follows a Geometric Brownian Motion: Merton derived the value of three assets among which the zero-coupon (but also the coupon-bearing and callable bonds)

$$dV_t = V_t([r - \delta]dt + \sigma dW_t)$$  \(2\)
1. The company issue a zero-coupon bond with face value $B$ and maturity $T$. 
Merton’s Model Breakthroughs

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2. Default occurs when the value $V_T$ of the asset is below the level of the debt $B$. 

**Figure:** Short Put on Assets
Merton’s Model Breakthroughs

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3. Default may occur only at date $T$ in which case the creditors take over the firm without incurring any distress costs and realize the amount $V_T$, so the payoff to the creditor at time $T$ is:

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\[
\begin{align*}
D(V_t, T) &= B - \text{Put}(V_t, B, r, R - t, \sigma) \\
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Put-call parity tells us that the shareholder holds a call:

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So at time $t$ the value of the bond and the stock are:

$$D(V_t, T) = B - \text{Put}(V_t, B, r, R - t, \sigma)$$
$$E(V_t, t) = \text{Call}(V_t, B, r, T - t, \sigma)$$ \hspace{1cm} (4)
As far as we know the diffusion process associated to the assets we can obtain the value of the debt and the equity of a given company through the Black, Scholes and Merton (1973) formula:

\[
C[V_t, B, r, T - t, \sigma] = C_t = N(d_1)V_t - N(d_2)Be^{-r(T-t)}
\]

\[
P[V_t, B, r, T - t, \sigma] = P_t = C_t + Be^{-r(T-t)} - V_t
\]

where:

\[
d_1 = \frac{\ln \left( \frac{V_t}{B} \right) + \left( r + \frac{\sigma^2}{2} \right) (T - t)}{\sigma \sqrt{T - t}}
\]

\[
d_2 = d_1 - \sigma \sqrt{T - t}
\]
We then get the probability of default $PD_i$ for any company $i$ as far as we manage to model the asset dynamic $A_t$.

**Figure:** Graph representation of the Merton’s Theory.
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**Figure:** Graph representation of the Merton’s Theory.

**Contribution of our work:** How can we model farms assets dynamic using price of the commodities and weather conditions?
For each farmer we know the breakdown of land use per type of crop.
Farm's Asset Modelling

Assumptions

- For each farmer we know the breakdown of land use per type of crop
- The asset value of a farm is a cumulative function of the farm profits (under the retained earnings mechanism)
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- For each farmer we know the breakdown of land use per type of crop.
- The asset value of a farm is a cumulative function of the farm profits (under the retained earnings mechanism).
- Conditional on the global filtration, we define an adapted process for the asset value of farm $i$ at time $t$:

$$A^i_t | \mathcal{F}_{t-1} = [A^i_{t-1} + R^i_t + \Delta E^i_t + \Delta D^i_t, | \mathcal{F}_{t-1}]$$  \hspace{1cm} (7)

If we assume that the farmer will not issue debt or equity from one year to another $\Delta E^i_t = 0$ and $\Delta D^i_t = 0$:

$$A^i_t | \mathcal{F}_{t-1} = \left[ A^i_{t-1} \left( 1 + \frac{R^i_t}{A^i_{t-1}} \right) \right] | \mathcal{F}_{t-1},$$  \hspace{1cm} (8)

where:

$$\left[ \frac{R^i_t}{A^i_{t-1}} \right] | \mathcal{F}_{t-1} = \sum_{k=1}^{K} \triangle^i_{k,t} [y^i_{k,t} (\tilde{\omega}^i_{\tau^i_k} \cdot \tilde{C}_{k,t}) | \mathcal{F}_{t-1}] - F^i_t,$$  \hspace{1cm} (9)
Assumptions

The vector \( \omega_t = \{T_t, P_t\} \in \mathbb{R}^S \times \mathbb{R}^S \) represents the information about weather conditions over time. Where \( T_t \) and \( P_t \) stand respectively for the temperature and the log-precipitation random variables associated to a set of \( S \) meteorological stations non-equally spread over a given territory.
Assumptions

- The vector $\omega_t = \{T_t, P_t\} \in \mathbb{R}^S \times \mathbb{R}^S$ represents the information about weather conditions over time. Where $T_t$ and $P_t$ stand respectively for the temperature and the log-precipitation random variables associated to a set of $S$ meteorological stations non-equally spread over a given territory.

- The filtration generated by the weather conditions $\{\omega_t\}_{t \geq 0}$ is denoted $\mathcal{H}_t$ while $\mathcal{Y}_t$ represents the filtration generated by the crop yields and we finally denoted the commodity prices vector $\{C_t\}_{t \geq 0} \in \mathbb{R}^K$ and its associated filtration $\mathcal{C}_t$ such that $\mathcal{F}_t = \mathcal{H}_t \vee \mathcal{C}_t \vee \mathcal{Y}_t$. 
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- the \( \mathcal{F}_t \)-measurable random variable \( R_{i,t} \) embodies the retained earning generated over the ending year by the farmer \( i \) and is function both of the weather conditions \( \omega_t \) through the \( K \) crop yields generated by the farmer \( i \) at time \( t \) and \( C_t \), the agricultural commodity price at which he sold his harvested or not yet harvested crops.
The conditional dynamic retained earnings process for farm $i$ at time $t$ is written as:

$$
\left[ \frac{R_t^i}{A_{t-1}^i} \right| \mathcal{F}_{t-1} = \sum_{k=1}^{K} \Delta_{k,t}^i [y_{k,t}^i (\tilde{\omega}_{k,t}^i) \cdot \tilde{C}_{k,t} | \mathcal{F}_{t-1}] - F_t^i, \quad (10)
$$
Farm’s Asset Modelling

Farms Profits Dynamic

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$$

- \( \triangle_{k,t}^{(i)} = \frac{\delta_{k,t}^i}{A^i_{t-1}} \) where \( \delta_{k,t}^i \) the hectares allocated by the farmer $i$ to the crop $k$
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- $\triangle_{(i)}^k = \frac{\delta^i_{k,t}}{A^i_{t-1}}$ where $\delta^i_{k,t}$ the hectares allocated by the farmer $i$ to the crop $k$

- $y^i_{k,t} \left( \tilde{\omega}^i_{\tau^k_t} \right)$ denotes the yield per hectares for a given crops and under given weather condition $\tilde{\omega}_t$ for the period of time $\tau^k_t$
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- $\tilde{C}_k,t = (\tilde{c}_k,t - v_{k,t}^i)$ represents the random price of a specific commodity $k$ on the market at a given time $t$

- $v_{k,t}^i$ represent the variable cost associated to the crop $k$

- $F_t^i = \frac{f_t^i}{A_{t-1}^i}$ for the fixed costs independent from the type of crop.
We can distinguish two sources of uncertainty:

- A local risk related to weather conditions: due to the relation between weather conditions and crops, bad weather conditions in a specific region doesn't necessarily impact other regions or other countries, leading to local dependence among farmers.

- A global risk related to market prices of agricultural commodities: due to the relation between these prices and the profits generated by farmers, commodity markets globalisation and transportation networks development link local prices to international market prices, generating a global dependence: a large price decrease of a given commodity may impact both the Romanian and the American farmers.
We can distinguish two sources of uncertainty:

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  - due to the relation between weather conditions and crops yield
  - bad weather conditions in a specific region doesn’t necessarily impact other region or other countries.
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We first analyze the conditional loss distribution given the filtration of the weather conditions $\mathcal{H}_t$ and the yield $\mathcal{Y}_t$:

- We assume as known and unchanged the yields associated to each farm.
Commodities Conditional Loss Distribution

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- We assume as known and unchanged the yields associated to each farm
- We only consider uncertainty about commodity prices
- We assume the following dynamic for the commodities market prices.

\[ d\tilde{C}_t = \tilde{\mu}_t dt + \tilde{\Omega}_t dW_t \]

where $dW_t$ is the vector of $dW^k_t$ associated to the $K$ $\mathcal{F}_t$-standard Brownian motion \( \{W^k_t\}_{t \geq 0} \), for $k = 1 \ldots K$. The matrix of variance covariance at time $t$ is then equal to $\tilde{\Omega}_t^T \tilde{\Omega}_t$ and

\[ d\tilde{C}_t \sim N(\tilde{\mu}_t dt, \tilde{\Omega}_t^T \tilde{\Omega}_t dt) \]
Commodities Conditional Loss Distribution

Assuming that the yield vector $\mathcal{Y}_t = y_{k,t}^i(\omega_t) \forall i, k$ is known, that $\tilde{C}_{k,t}$ is independent from the local weather conditions, we can then rewrite the previous equation given

$$E \left( \frac{R_t^i}{A_{t-1}^i} \mid \mathcal{H}_t, \mathcal{Y}_t \right) = H_{i,t}(\omega_t) \left( \tilde{C}_{t-1} + \tilde{\mu}_t \Delta t \right) - F_t^i$$

with:

$$H_{i,t}(\omega_t) = \begin{bmatrix} \Delta_{1,t}^i y_{1,t}^i(\omega_t) \\ \vdots \\ \Delta_{K,t}^i y_{K,t}^i(\omega_t) \end{bmatrix}$$

and:

$$\tilde{C}_{t-1} = \left( \tilde{C}_{1,t-1}, \ldots, \tilde{C}_{K,t-1} \right)^T$$
Commodities Conditional Loss Distribution

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$$\mathbb{E}\left(\frac{R_i^t}{A_{i-1}^t} | \mathcal{H}_t, \mathcal{Y}_t\right) = H_{i,t}(\omega_t) \left(\tilde{C}_{t-1} + \tilde{\mu}_t \Delta t\right) - F_i^t$$

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$$\tilde{C}_{t-1} = \left(\tilde{C}_{1,t-1}, \ldots, \tilde{C}_{K,t-1}\right)^\top$$

- We can also express the profits conditional variance as follows:

$$\sigma_{i,t}^2 | \mathcal{H}_t, \mathcal{Y}_t = \mathbb{V}\left(\frac{R_i^t}{A_{i-1}^t} | \mathcal{H}_t, \mathcal{Y}_t\right) = H_{i,t}(\omega_t) \tilde{\Omega}_t^\top \tilde{\Omega}_t H_{i,t}(\omega_t)^\top$$
The risk of default of the farmer $i$ is then expressed as follows:

$$PD_i | \mathcal{H}_t, \mathcal{Y}_t = Pr \left[ A_{i,t} \leq D_{i,t} | \mathcal{H}_t, \mathcal{Y}_t \right]$$

$$= Pr \left[ \frac{R_{i,t}}{A_{t-1}'} \leq \frac{D_{i,t}}{A_{t-1}'} - 1 | \mathcal{H}_t, \mathcal{Y}_t \right]$$

$$= \Phi \left[ \frac{\left( \frac{D_{i,t}}{A_{t-1}'} - 1 \right) - H_{i,t}(\omega_t)(\tilde{C}_{t-1} + \tilde{\mu}_t) + F_t^i}{\sqrt{H_{i,t}(\omega_t)\tilde{\Omega}_t^\top \tilde{\Omega}_t H_{i,t}(\omega_t)^\top}} \right] | \mathcal{H}_t, \mathcal{Y}_t$$
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$$= \Phi \left[ \left( \frac{D_{i,t}}{A'_{t-1}} - 1 \right) - \mathcal{H}_{i,t}(\omega_t)(\tilde{C}_{t-1} + \tilde{\mu}_t) + F^i_t \right]$$

$$\sqrt{\mathcal{H}_{i,t}(\omega_t)\tilde{\Omega}_t^\top \tilde{\Omega}_t \mathcal{H}_{i,t}(\omega_t)^\top} | \mathcal{H}_t, \mathcal{Y}_t$$

While the default correlation between the farmer $i$ and farmer $j$ can be computed under the assumption of Gaussian joint distribution such as:

$$\rho_{ij}|\mathcal{H}_t, \mathcal{Y}_t = \frac{Pr \left( A_{i,t} \leq D_{i,t}, A_{j,t} \leq D_{j,t} | \mathcal{H}_t, \mathcal{Y}_t \right) - PD_i|\mathcal{H}_t, \mathcal{Y}_t PD_j|\mathcal{H}_t, \mathcal{Y}_t}{\sqrt{PD_i|\mathcal{H}_t, \mathcal{Y}_t (1 - PD_i|\mathcal{H}_t, \mathcal{Y}_t) PD_j|\mathcal{H}_t, \mathcal{Y}_t (1 - PD_j|\mathcal{H}_t, \mathcal{Y}_t)}}$$

where:

$$Pr \left( \frac{R_{i,t}}{A'_{t-1}} \leq \frac{D_{i,t}}{A'_{t-1}} - 1, \frac{R_{j,t}}{A'_{t-1}} \leq \frac{D_{j,t}}{A'_{t-1}} - 1 | \mathcal{H}_t, \mathcal{Y}_t \right)$$

$$= \int_0^{\frac{D_{i,t}}{A'_{t-1}} - 1} \int_0^{\frac{D_{j,t}}{A'_{t-1}} - 1} MVN \left( \frac{R_{i,t}}{A'_{t-1}}, \frac{R_{j,t}}{A'_{t-1}}, \theta_{ij} | \mathcal{H}_t, \mathcal{Y}_t \right) dR_{j,t} dR_{i,t}$$
with:

$$\theta_{ij|H_t, Y_t} = \frac{\text{cov}(R_i, R_j|H_t, Y_t)}{\sqrt{\text{V}(R_i, t|H_t, Y_t) \text{V}(R_j, t|H_t, Y_t)}}$$

while:

$$\text{cov}(R_i, R_j|H_t, Y_t) = H_{i,t}(\omega_t)\tilde{\Omega}_t^\top \tilde{\Omega}_t H_{j,t}(\omega_t)^\top$$

and:

$$\text{MVN} \left(R_{i,t}, R_{j,t}, \theta_{ij|H_t, Y_t} | H_t, Y_t \right) = \frac{1}{2\pi \sigma_{i,t} \sigma_{j,t} \sqrt{1-(\theta_{ij|H_t, Y_t})^2}} \exp \left\{ \frac{-1}{2(1-(\theta_{ij|H_t, Y_t})^2)} \left\{ \frac{\left(R_{i,t}-H_{i,t}(\omega_t)(\tilde{c}_{t-1}+\bar{\mu}_t)+F_{i,t}\right)^2}{\sqrt{\text{V}(R_{i,t}|H_t, Y_t)}} + \frac{\left(R_{j,t}-H_{j,t}(\omega_t)(\tilde{c}_{t-1}+\bar{\mu}_t)+F_{j,t}\right)^2}{\sqrt{\text{V}(R_{j,t}|H_t, Y_t)}} \right\} \right\}$$
We can then compute respectively the portfolio loss function $L$ of the farms’ creditor, the expected loss $EL$ and the unexpected loss $UL$ which respectively represents the expected value and the variance of the loss function associated to this portfolio of loans.

$$L|H_t, Y_t = \sum_{n=1}^{N} EAD_n LGD_n D_n|H_t, Y_t$$

where $D_n|H_t, Y_t \sim Bernoulli\left(PD_n|H_t, Y_t\right)$. In order to simplify the forthcoming notations we use $PD_n^* = PD_n|H_t, Y_t$
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$$EL|\mathcal{H}_t, \mathcal{Y}_t = \sum_{n=1}^{N} EAD_nELGD_nPD_n^*$$

$$UL|\mathcal{H}_t, \mathcal{Y}_t = \sqrt{V(L|\mathcal{H}_t, \mathcal{Y}_t)}$$

$$= \sqrt{\sum_{n,k=1}^{N} EAD_nEAD_kELGD_nELGD_k\rho_{nk}\sqrt{PD_n^*(1-PD_n^*)}} PD_n^*(1-PD_n^*)$$
Weather Conditional Loss Distribution

Roadmap:

- To take into consideration the impact of the weather condition on the profit of the farmers and thus on their probability of default we consider the yield associated to each farm as a function of local weather conditions.
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- We model the yield of each farm as a linear function of non linear estimator of temperatures and precipitations associated to this region.

- In order to obtain the temperature and precipitation estimators for all the farms according to their respective longitude and latitude we consider a gaussian process model with as design points for the input space a set of weather stations records.
Crop yield distribution modelling for calculating an insurance premium.

- **Parametric:** Nelson and Preckel, 1989 applied a conditional beta distribution to study corn yields modelling notwithstanding but difficult to obtain the standard errors of moment elasticities.
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- Non-parametric methods offer an appealing flexibility since they heavily rely on the data sample to determine the most appropriate density representation avoiding thus the restraining choice of a specific parametric probability distribution, their rate of convergence to the true distribution might be relatively slow and consequently makes those methods data-intensive. (Sherrick et al., 2014)
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- More recently, a couple of articles investigate instead the yield distribution at the farm level in order to get a better grasp on the aggregation process to the county level (Gerlt et al. 2014, Claassen and Just 2011).
Multivariate weather time series associated to a set of weather stations geolocalisations not only encompasses spatial dependence structure but also temporal relationships.
Time and Space Decomposition

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  - a national (or a county) global seasonality-adapted trend $\bar{w}_t$ evenly impacting all the country regions and model through a SARIMA model
  - a spatial dependence structure which furnishes a local adjustment for each weather station through a spatial Gaussian Process $f^w(x)$
**Weather Risk**

\[ W_T^T(x) \] representing the observed precipitation \( P_t \) or temperature \( T_t \) are defined as follows:

\[
\widetilde{W}_t^T(x) = W_t^T(x) - \mathbb{E}_t \left[ \mathbb{E}_x \left( W_t^T(x) \right) \ | w_{t-1}, w_{t-2}, \ldots \right] 1_n
\]  

\[
= f^T(x, t) + \epsilon_{t,x}
\]

with \( \epsilon_{t,x} \sim \mathcal{N} \left( 0, \sigma^2_t 1_n \right) \)
Time and Space Decomposition

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\]

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with \( \epsilon_{t,x} \sim \mathcal{N}(0, \sigma_t^2 1_n) \)

- where the integrated and seasonally adjusted conditional mean operator \( E_t \) copes with the serial correlation observed in temperature and precipitation data at the level of the country through a SARIMA\((p, d, q)(P, D, Q)_s\) trend formulation:

\[
\Phi_P(B^s) \cdot \phi(B) \cdot \nabla_s^D \cdot \nabla^d \cdot X_t = c + \Theta_Q(B^s) \cdot \theta(B) \cdot \varepsilon_t.
\]

where the integer \( p, d \) and \( q \) is refereed respectively to the order of autoregression, of integration and the number of moving average lags. \( B^k X_t = X_{t-k} \) represents the backshift operator. While:

\[
\tilde{\varepsilon}_t \sim \text{iid } \mathcal{N}(0, \sigma_{\tilde{\varepsilon}}^2)
\]
To impose weak stationarity to the discrete-time stochastic process required that $\phi(B) \neq 0$ and $\phi(B)$ has all roots outside unit disc. $\nabla^D_s$ and $\nabla^d$ denote respectively the seasonal difference and non-seasonal difference components.
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Furthermore, the spatial conditional expected value $\mathbb{E}_x$ which corresponds to the average value of the temperatures collected by all the weather stations on a given date $t$:

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\mathbb{E}_x \left( W^T_t(x) \right) = n^{-1} \sum_{i=1}^{n} w^T_t(x_i)
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Furthermore, the **spatial conditional expected value** $\mathbb{E}_x$ which corresponds to the average value of the temperatures collected by all the weather stations on a given date $t$:

$$\mathbb{E}_x \left( W_t^T(x) \right) = n^{-1} \sum_{i=1}^{n} w_t^T(x_i)$$

while the **spatial Gaussian Process** $f^w(x)$ is defined such as:

$$f^T(x, t) \sim \mathcal{G}\mathcal{P} \left( 0, k(x, x^*; t, t^*) \right)$$

with $w_t^T = \{w_t^T(x_1), w_t^T(x_2), \ldots, w_t^T(x_n)\}$ denotes the vector of the temperature observed for the $n$ weather stations with the associated $n$ locations vectors written as $x = \{x_1, x_2, \ldots, x_n\}$. 
Two Types of Gaussian Process

Spatial Kernel

- The Gaussian Process is fully specified by a conditional mean function \( \mu(x) \) and a conditional covariance function which we consider time independent \( k(x, x^*) \) such that:

\[
\begin{align*}
  f^T(x, t) &\sim \mathcal{GP}(\mu(x), k(x, x^*)) \\
  \mu(x) &= \mathbb{E}[f(x)] \\
  k(x, x^*) &= \mathbb{E}[(f(x) - \mu(x))(f(x^*) - \mu(x^*))]
\end{align*}
\]

where \( x \) and \( x^* \) represent two different location vectors.
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$$k(x, x^*) = \mathbb{E}[(f(x) - \mu(x)) (f(x^*) - \mu(x^*))]$$

where $x$ and $x^*$ represent two different location vectors.

- For the purpose of this paper, we assume the random variables $f_t(x)$ associated to the location vector $x = \{x^{Lg}, x^{Lt}\}$ to be characterised by a zero-mean and the following covariance function:

$$k(x, x^*) = \text{cov}(f_t(x), f_t(x^*)) = (\sigma^f_t)^2 \exp\left[-\frac{1}{2} (x - x^*)^\top M_t (x - x^*)\right]$$

which corresponds to the squared exponential covariance function and is fully specified by the hyperparameter $\sigma^f_t$ and the symmetric matrix $M_t = \text{diag}(\theta_t)^{-1}$, where $\theta_t = \{\theta^{Lg}_t, \theta^{Lt}_t\}$ corresponds to the vector of the longitude and latitude scaling hyperparameters.
If we assume that the gaussian process as zero-mean \( \mu(x) = 0 \) so that 
\[
\tilde{W}_t^T(x) \sim \mathcal{N}(0, K + \sigma_t^2 I_n)
\]
where \( K = (\sigma_t)^2 K' \) and the correlation \( K' \) having elements \( k(x_i, x_j) \) we can write then the marginal likelihood such as:

\[
\log p(\tilde{W}_t^T(x) | X) = -\frac{1}{2} \tilde{W}_t^T(x)^\top (K + \sigma_t^2 I_n)^{-1} \tilde{W}_t^T(x) - \frac{1}{2} \log |K + \sigma_t^2 I_n| - \frac{n}{2} \log 2\pi
\]
Two Types of Gaussian Process

Estimation Procedure

If we assume that the gaussian process as zero-mean \( \mu(x) = 0 \) so that \( \tilde{W}_t^T(x) \sim \mathcal{N}(0, K + \sigma_t^2 I_n) \) where \( K = (\sigma_f)^2 K' \) and the correlation \( K' \) having elements \( k(x_i, x_j) \) we can write then the marginal likelihood such as:

\[
\log p(\tilde{W}_t^T(x)|X) = -\frac{1}{2} \tilde{W}_t^T(x)^\top (K + \sigma_t^2 I_n)^{-1} \tilde{W}_t^T(x) - \frac{1}{2} \log |K + \sigma_t^2 I_n| - \frac{n}{2} \log 2\pi
\]

To set the hyperparameters by maximizing the marginal likelihood w.r.t. the hyperparameters such that:

\[
\frac{\partial}{\partial \theta_j} \log p(\tilde{W}_t^T(x)|X, \theta) = \frac{1}{2} \tilde{W}_t^T(x)^\top K_\tilde{W}^{-1} \frac{\partial K}{\partial \theta_j} K_\tilde{W}^{-1} \tilde{W}_t^T(x) - \frac{1}{2} \text{tr}(K_\tilde{W} \frac{\partial K}{\partial \theta_j})
\]

\[
= \frac{1}{2} \text{tr}((\alpha \alpha^\top - K_\tilde{W}^{-1}) \frac{\partial K}{\partial \theta_j})
\]

where \( \alpha = K_\tilde{W}^{-1} \tilde{W}_t^T(x) \) and \( K_\tilde{W} = K + \sigma_t^2 I_n \)
Local Approximation GP

- A laGP is a localized approximated emulation by means of a fast sequential updating greedy search algorithm in order to minimize the mean-squared prediction error (MSPE).
Local Approximation GP

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- The idea is to remove some vanishingly low impact observed sites while maintain the rest of the reference points under certain criteria, including active learning Cohn (ALC) and MSPE.
Two Types of Gaussian Process

Local Approximation GP

- A laGP is a localized approximated emulation by means of a fast sequential updating greedy search algorithm in order to minimize the mean-squared prediction error (MSPE).
- The idea is to remove some vanishingly low impact observed sites while maintain the rest of the reference points under certain criteria, including active learning Cohn (ALC) and MSPE.
- The iterative estimation starts from a small subset $D_{n_0}(x) = (X_{n_0}(x), Y_{n_0}(x))$ close to $x$ and to choose $x_{j+1}$ to augment $X_j(x)$ and thus form a new subset $D_{j+1}(x)$ according to the MSPE objective criteria to minimize which is defined as:

$$J(x_{j+1}, x) = \mathbb{E} \left\{ \left[ Y(x) - \mu_{j+1}(x; D_{j+1}(x), \hat{\theta}_{j+1}) \right]^2 \mid D_j(x) \right\}$$

which can be approximated by:

$$J(x_{j+1}, x) \approx V_j(x|x_{j+1}; \hat{\theta}_j) + \left( \frac{\partial \mu_j(x; \theta)}{\partial \theta} \bigg|_{\theta=\hat{\theta}_j} \right)^2 / I_{j+1}(\hat{\theta}_j),$$

where $I$ is the expected Fisher information.
Two Types of Gaussian Process

Local Approximation GP

Which is equivalent to:

$$\text{argmax}_{x_{j+1} \in x \setminus x_j} \{ V_j (x; \theta) - V_{j+1} (x; \theta) \}. \quad (15)$$
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with:

$$\mu(x) = \left[ \frac{1}{1_n^T R^{-1} 1_n} 1_n^T + r^T \right] R^{-1} y_t,$$

where $r$ is the vector of correlations between the input $x$ and $x_{i=1,...,n}$ at the $n$ design sites, $r = [ \text{cor} (f(x_1), f(x)), \ldots, \text{cor} (f(x_n), f(x)) ]$. While the mean squared error (MSE) is expressed such as follows:

$$V_j (x; \theta) = (\delta_t^f)^2 \cdot \left( 1 - r^T R^{-1} r + \frac{(1 - 1_n^T R^{-1} 1_n)^2}{1_n^T R^{-1} 1_n} \right).$$
Two Types of Gaussian Process

Local Approximation GP

- Which is equivalent to:

\[
\arg\max_{x_{j+1} \in \mathbf{x} \setminus x_j} \{ V_j (\mathbf{x}; \theta) - V_{j+1} (\mathbf{x}; \theta) \}.
\]  

(15)

- with:

\[
\mu (\mathbf{x}) = \left[ \left( \frac{1 - r^\top R^{-1} 1_n}{1_n^\top R^{-1} 1_n} \right) 1_n^\top + r^\top \right] R^{-1} \mathbf{y}_t,
\]

where \( r \) is the vector of correlations between the input \( \mathbf{x} \) and \( \mathbf{x}_{i=1,...,n} \) at the \( n \) design sites, \( r = [\text{cor} \left(f(\mathbf{x}_1), f(\mathbf{x}))\right), \ldots, \text{cor} (f(\mathbf{x}_n), f(\mathbf{x}))]. \) While the mean squared error (MSE) is expressed such as follows:

\[
V_j (\mathbf{x}; \theta) = (\hat{\sigma}^f_t)^2 \cdot \left( 1 - r^\top R^{-1} r + \left( \frac{1 - 1_n^\top R^{-1} 1_n}{1_n^\top R^{-1} 1_n} \right)^2 \right).
\]

- We then update the subset to \( D_{j+1}(\mathbf{x}) \) meanwhile independently compute the hyper-parameter \( \hat{\theta}_j(\mathbf{x}) | D_j(\mathbf{x}) \) by maximizing the likelihood which possibly could smooth spatially over all the locations.
Yield Model

We denote \( \hat{y}_{k,t}^i \left( \tilde{\omega}_{t_k}^i \right) \) as the best linear unbiased predictor of the yield per hectares at time \( t \) of the farm \( i \) for the \( k \)-th crop and function of the random variable \( \tilde{\omega}_{t_k}^i \) which embodies the precipitation and temperature over the period \( \tau_t^k \) (Bokusheva, 2014; Roberts and al., 2012) as:

\[
\hat{y}_{k,t}^i \left( \tilde{\omega}_{t_k}^i \right) = \alpha_0 + \alpha_{P,k} \cdot \hat{W}_{k,t_k}^P + \alpha_{T,k} \cdot \hat{W}_{k,t_k}^T,
\]

(16)
The Yield Model

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\[
\hat{y}_{k,t}^{i} \left( \tilde{\omega}_{\tau_{k}^{t}}^{i} \right) = \alpha_{0} + \alpha_{P,k} \cdot \hat{W}_{k,\tau_{k}^{t}}^{P,i} + \alpha_{T,k} \cdot \hat{W}_{k,\tau_{k}^{t}}^{T,i},
\]

(16)

- If we substitute the unbiased out-of-sample predictive value of weather random variables, we will then have:

\[
\hat{y}_{k,t}^{i} \left( \tilde{\omega}_{\tau_{k}^{t}}^{i} \right) = \alpha_{0} + \alpha_{P,k} \cdot \left( \bar{\omega}_{\tau_{k}^{t}}^{P} + \hat{f}^{P}(x_{i}) \right) + \alpha_{T,k} \cdot \left( \bar{\omega}_{\tau_{k}^{t}}^{T} + \hat{f}^{T}(x_{i}) \right)
\]

where \( \alpha_{0} \) is constant and \( \left( y_{k,t}^{i} - \hat{y}_{k,t}^{i} \left( \tilde{\omega}_{\tau_{k}^{t}}^{i} \right) \right) \sim N \left( 0, \Psi_{\tau_{k}^{t}}^{i} \right) \) while:

\[
\bar{\omega}_{t}^{T} = E_{t} \left[ E_{x} \left( W_{t}^{T}(x) \right) \middle| w_{t-1}^{T}, w_{t-2}^{T}, \ldots \right]
\]

corresponds to the SARIMA expected temperature at the country level. While for the precipitations we have the same expression:

\[
\bar{\omega}_{t}^{P} = E_{t} \left[ E_{x} \left( W_{t}^{P}(x) \right) \middle| w_{t-1}^{P}, w_{t-2}^{P}, \ldots \right]
\]
The Yield Model

Weather Conditional Loss Distribution

Assuming that \( \tilde{\omega}_t^i = \{ \tilde{W}_{k,t}^T, \tilde{W}_{k,t}^P \} \) are both independently and identically normally distributed leads to the farm-\( i \)'s expected yield \( \hat{y}_{k,t}^i (\tilde{\omega}_{\tau_t}^i) \) with a variance equals to the \( i \)-th element on the diagonal of the variance covariance matrix:

\[
\text{Var} \left( \mathbf{y}_{k,t} \left( \tilde{\omega}_{\tau_t}^i \right) \right) = \alpha_{P,k}^2 \cdot \text{Var} \left( \tilde{W}_t^P (\mathbf{x}) \right) + \alpha_{T,k}^2 \cdot \text{Var} \left( \tilde{W}_t^T (\mathbf{x}) \right) + \Psi_{\tau_t}^i
\]

where:

\[
\text{Var} \left( \tilde{W}_t^T (\mathbf{x}) \right) = \text{Var} \left[ \mathbb{E}_\mathbf{x} \left( \mathbf{W}_t^T (\mathbf{x}) \right) | \mathbf{w}_{t-1}^T, \mathbf{w}_{t-2}^T, \ldots \right] \mathbf{I}_n + K(\mathbf{x}, \mathbf{x}) + \sigma_t^2 \mathbf{I}_n,
\]

While \( \text{Var} \left[ \mathbb{E}_\mathbf{x} \left( \mathbf{W}_t^T (\mathbf{x}) \right) | \mathbf{w}_{t-1}^T, \mathbf{w}_{t-2}^T, \ldots \right] \) can be derived from \( \Gamma(\mathbf{x}) \), the autocovariance generating function (AGF) which for summable autocovariance functions \( \sum_{h=-\infty}^{\infty} \gamma(h) < \infty \) is defined such that:

\[
\Gamma(\mathbf{x}) = \sum_{h=-\infty}^{\infty} \gamma(h) \mathbf{x}^h \tag{17}
\]

where \( \gamma(h) \) is the process autocovariance between \( x_t \) and \( x_{t+h} \).
We can then determine another conditional probability of default which is linked now to the weather conditions $\omega_t$ and their local impact conditionally on the $K$ net variable prices of the commodities $\tilde{C}_t = (\tilde{C}^1_t, \ldots, \tilde{C}^K_t)$. 
The Yield Model

Weather Conditional Loss Distribution

- We can then determine another conditional probability of default which is linked now to the weather conditions $\omega_t$ and their local impact conditionally on the $K$ net variable prices of the commodities $\tilde{C}_t = (\tilde{C}_t^1, \ldots, \tilde{C}_t^K)$.

- Using the properties of the Gaussian process we can then write the value of the conditional expected returns conditionally on the commodity prices filtration $C_t$:

$$
E \left( \frac{R_{i,t}}{A_{i,t-1}} \mid \mathcal{H}_t, C_t \right) = \sum_{k=1}^K \Delta_{k,t}^i E \left[ y_{k,t}^i \left( \tilde{\omega}_{k,t}^i \right) \mid \tilde{C}_k, t \mid \mathcal{H}_t, C_t \right] - F_t^i
$$

$$
= \sum_{k=1}^K \Delta_{k,t}^i E \left[ y_{k,t}^i \left( \tilde{\omega}_{k,t}^i \right) \mid \mathcal{H}_t, C_t \right] \tilde{C}_k, t - F_t^i
$$

$$
= \begin{bmatrix}
\Delta_{1,t}^i E \left[ y_{1,t}^i \left( \tilde{\omega}_{1,t}^i \right) \mid \mathcal{H}_t, C_t \right] \\
\vdots \\
\Delta_{K,t}^i E \left[ y_{K,t}^i \left( \tilde{\omega}_{K,t}^i \right) \mid \mathcal{H}_t, C_t \right]
\end{bmatrix}
- F_t^i
$$
The Yield Model

While the log-return variance conditionally on the $K$ net variable prices of the commodities $\tilde{C}_t = (\tilde{C}_t^1, \ldots, \tilde{C}_t^K)$ is given by:

$$\text{Var} \left( \frac{R_{i,t}}{A_{t-1}^i} | H_t, C_t \right) = \tilde{C}_t^T \begin{bmatrix} (\Delta_{1,t}^i)^2 \text{Var} \left[ y_{1,t}^i \left( \tilde{\omega}_{\tau_k}^i \right) | H_t \right] \\ \vdots \\ (\Delta_{K,t}^i)^2 \text{Var} \left[ y_{K,t}^i \left( \tilde{\omega}_{\tau_k}^i \right) | H_t \right] \end{bmatrix} \tilde{C}_t$$
While the log-return variance conditionally on the $K$ net variable prices of the commodities $\tilde{C}_t = (\tilde{C}^1_t, \ldots, \tilde{C}^K_t)$ is given by:

$$
\nabla \left( \frac{R_{i,t}}{A_{i,t-1}} \mid \mathcal{H}_t, C_t \right) = \tilde{C}_t^\top \begin{bmatrix}
(D_{1,t})^2 \nabla [y^i_{1,t} \left( \tilde{w}^i_{\tau_k,t} \right) \mid \mathcal{H}_t] \\
\vdots \\
(D_{K,t})^2 \nabla [y^i_{K,t} \left( \tilde{w}^i_{\tau_k,t} \right) \mid \mathcal{H}_t]
\end{bmatrix} \tilde{C}_t
$$

Eventually the local probability of default of the farmer $i$ can be expressed such as:

$$
PD_i \mid \mathcal{H}_t, C_t = Pr \left[ A_{i,t} \leq D_{i,t} \mid \mathcal{H}_t, C_t \right] \\
= Pr \left[ \frac{R_{i,t}}{A_{i,t-1}^i} \leq \frac{D_{i,t}}{A_{i,t-1}} - 1 \mid \mathcal{H}_t, C_t \right] \\
= \Phi \frac{\nabla \left( \frac{R_{i,t}}{A_{i,t-1}^i} \mid \mathcal{H}_t, C_t \right)}{\sqrt{\nabla \left( \frac{R_{i,t}}{A_{i,t-1}^i} \mid \mathcal{H}_t, C_t \right)}}
$$
The default correlation between the farmer \( i \) and farmer \( j \) can naturally be computed under the assumption of Gaussian joint distribution such as:

\[
\rho_{ij|H_t, C_t} = \frac{Pr \left( A_{i,t} \leq D_{i,t}, A_{j,t} \leq D_{j,t} | H_t, C_t \right) - PD_{i|H_t, C_t} PD_{j|H_t, C_t}}{\sqrt{PD_{i|H_t, C_t} (1 - PD_{i|H_t, C_t}) PD_{j|H_t, C_t} (1 - PD_{j|H_t, C_t})}}
\]

where:

\[
Pr \left( \frac{R_{i,t}}{A_{i,t-1}} \leq \frac{D_{i,t}}{A_{i,t-1}} - 1, \frac{R_{j,t}}{A_{j,t-1}} \leq \frac{D_{j,t}}{A_{j,t-1}} - 1 | H_t, C_t \right) = \int_{0}^{D_{i,t}} \int_{0}^{D_{j,t}} \text{MVN} \left( \frac{R_{i,t}}{A_{i,t-1}}, \frac{R_{j,t}}{A_{j,t-1}}, \theta_{ij|H_t, C_t} \right) dR_{j,t} dR_{i,t}
\]

with:

\[
\theta_{ij|H_t, C_t} = \frac{\text{Cov}(R_{i,t}, R_{j,t} | H_t, C_t)}{\sqrt{\text{V}(R_{i,t} | H_t, C_t) \text{V}(R_{j,t} | H_t, C_t)}}
\]
Weather Conditional Loss Distribution

while:

\[
\text{Cov}(R_i, R_j | \mathcal{H}_t, C_t) = \tilde{C}_t^T \begin{bmatrix}
\Delta_{1,t} \text{Cov} \left[ y_{1,t}^i \left( \tilde{\omega}_{\tau_t}^i \right), y_{1,t}^j \left( \tilde{\omega}_{\tau_t}^j \right) | \mathcal{H}_t, \right] \Delta_{1,t}^i \\
\vdots \\
\Delta_{K,t} \text{Cov} \left[ y_{K,t}^i \left( \tilde{\omega}_{\tau_t}^i \right), y_{K,t}^j \left( \tilde{\omega}_{\tau_t}^j \right) | \mathcal{H}_t, \right] \Delta_{K,t}^j
\end{bmatrix} \tilde{C}_t
\]

where:

\[
\text{Cov} \left[ y_{1,t}^i \left( \tilde{\omega}_{\tau_t}^i \right), y_{1,t}^j \left( \tilde{\omega}_{\tau_t}^j \right) | \mathcal{H}_t, \right] = \nabla \left( y_{k,t} \left( \tilde{\omega}_{\tau_t}^k \right) \right)_{i,j}
\]

with

\[
\nabla \left( y_{k,t} \left( \tilde{\omega}_{\tau_t}^k \right) \right) = \alpha_{p,k}^2 \nabla \left( \tilde{W}_t^P(x) \right) + \alpha_{T,k}^2 \nabla \left( \tilde{W}_t^T(x) \right) + \psi_{\tau_t}^i
\]

and

\[
\nabla \left( \tilde{W}_t^T(x) \right) = \nabla \left[ \mathbb{E}_x \left( W_t^T(x) \right) | w_{t-1}, w_{t-2}, \ldots \right] l_n + K(x, x) + \sigma_t^2 l_n,
\]
Weather Conditional Loss Distribution

and:

$$\text{MVN} \left( R_{i,t}, R_{j,t}, \theta_{ij} | \mathcal{H}_t, \mathcal{C}_t \right) = \frac{1}{2\pi \sqrt{\text{V}(R_{i,t} | \mathcal{H}_t, \mathcal{C}_t) \text{V}(R_{j,t} | \mathcal{H}_t, \mathcal{C}_t) \left(1 - (\theta_{ij} | \mathcal{H}_t, \mathcal{C}_t)^2\right)}}$$

$$\times \exp \left\{ \frac{-1}{2 \left(1 - (\theta_{ij} | \mathcal{H}_t, \mathcal{C}_t)^2\right)} \left( \frac{(R_{i,t} - E(R_{i,t} | \mathcal{H}_t, \mathcal{C}_t))^2}{\text{V}(R_{i,t} | \mathcal{H}_t, \mathcal{C}_t)} + \frac{(R_{j,t} - E(R_{j,t} | \mathcal{H}_t, \mathcal{C}_t))^2}{\text{V}(R_{j,t} | \mathcal{H}_t, \mathcal{C}_t)} \right) \right\}$$

we can then compute respectively the portfolio loss function $L$, the expected loss $EL$ and the unexpected loss $UL$ which respectively represents the expected value and the variance of the loss function associated to this portfolio of loans conditionally to the filtrations $\mathcal{C}_t$ and $\mathcal{H}_t$.

$$L | \mathcal{H}_t, \mathcal{C}_t = \sum_{n=1}^{N} \text{EAD}_n \text{LGD}_n D_n | \mathcal{H}_t, \mathcal{C}_t$$

where $D_n | \mathcal{H}_t, \mathcal{C}_t \sim \text{Bernoulli} \left( PD_n | \mathcal{H}_t, \mathcal{C}_t \right)$

$$EL | \mathcal{H}_t, \mathcal{C}_t = \sum_{n=1}^{N} \text{EAD}_n \text{ELGD}_n PD_n | \mathcal{H}_t, \mathcal{C}_t$$
The Yield Model

Weather Conditional Loss Distribution

\[ UL|H_t, C_t = \sqrt{V(L|H_t, C_t)} \]

\[ = \sqrt{\sum_{n,k=1}^{N} EAD_n EAD_k ELGD_n ELGD_k \rho_{nk} \sqrt{PD_n|H_t, C_t (1 - PD_n|H_t, C_t) PD_k|H_t, C_t (1 - PD_k|H_t, C_t)}} \]

(18)
The Yield Model

Farm’s Return Distribution

Considering the initial equation in matrix form:

\[
\begin{bmatrix} R_t \circ A_{t-1}^{o-1} \mid F_{t-1} \end{bmatrix} = \begin{bmatrix} \Delta_t \circ y_t(\tilde{\omega}) \cdot \tilde{C}_{k,t} \mid F_{t-1} \end{bmatrix} - F_t,
\]

(19)

The global risk being the product of two MVN distributions we obtain a unique MVN distribution with expected value:

\[
\mu_R = \Sigma_R \left( \Sigma_y^{-1} \mu_y + \Sigma_C^{-1} \mu_C \right)
\]

(20)

and a variance equals to:

\[
\Sigma_R = \left( \Sigma_y^{-1} + \Sigma_C^{-1} \right)^{-1}
\]

(21)

with a normalizing constant:

\[
\Sigma_R = (2\pi)^{-n/2} |\Sigma_y + \Sigma_C|^{-1/2} \exp \left( -\frac{1}{2} (\mu_y - \mu_R)^\top (\Sigma_y + \Sigma_C) (\mu_y - \mu_R) \right)
\]

(22)
1. Introduction

2. Credit Risk
   - Accounting basics
   - Merton’s Model
   - Farm’s Asset Modelling

3. Commodity Risk

4. Weather Risk
   - Review of the Literature
   - Time and Space Decomposition
   - Two Types of Gaussian Process
   - The Yield Model

5. Data and Results
   - Data Description
   - Models Comparison
   - Results

6. Conclusion
Real data obtained from a French fertilizer company, the Roullier Group.
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- type of crops,
- crops rotation,
- number of hectares cultivated
- a precise geolocalisation of each farm.

This farms sample adds up to 4.6 million hectares which occupied over one-third of the total Romanian utilized agricultural area (UAA)
Utilized Agricultural Area

Utilized agricultural area (UAA) in EU:
(40.0%) of the total land area of the EU-28 in 2013
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(40.0%) of the total land area of the EU-28 in 2013

1. France with 27,8 million hectares (16%)
2. Spain, with 23,75 million hectares (13,6%)
3. United Kingdom, with 16,88 million hectares (9,7%)
4. Germany, with 16,7 million hectares (9,6%)
5. Poland, with 14,4 million hectares (8,3%)
Utilized agricultural area (UAA) in EU: (40.0%) of the total land area of the EU-28 in 2013

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6. Romania holds 7.6% of the, with 13.05 million hectares...
Utilised agricultural area by land use:

- Cereals: 33.2%
- Other arable land: 26.5%
- Pasture and meadow: 21.9%
- Rough grazing: 11.4%
- Permanent crops: 5.9%
- Permanent grassland and meadow not used for production, eligible for subsidies: 0.8%
- Kitchen gardens: 0.2%

(1) Estimates. Source: Eurostat (online data code: el oluft)
# Data Description

## Agricultural Output Breakdown

<table>
<thead>
<tr>
<th>Output components</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>% of total</th>
<th>% of EU-28</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cereals:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wheat and spelt</td>
<td>1,351</td>
<td>1,252</td>
<td>1,295</td>
<td>13.7%</td>
<td>4.9%</td>
</tr>
<tr>
<td>Rye and meslin</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>0.0%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Barley</td>
<td>357</td>
<td>341</td>
<td>317</td>
<td>3.4%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Oats and summer cereal mixtures</td>
<td>93</td>
<td>81</td>
<td>65</td>
<td>0.7%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Grain maize</td>
<td>2,638</td>
<td>2,125</td>
<td>1,453</td>
<td>15.4%</td>
<td>16.5%</td>
</tr>
<tr>
<td>Rice</td>
<td>15</td>
<td>10</td>
<td>12</td>
<td>0.1%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Other cereals</td>
<td>42</td>
<td>38</td>
<td>37</td>
<td>0.4%</td>
<td>1.9%</td>
</tr>
<tr>
<td><strong>Industrial crops:</strong></td>
<td>1,238</td>
<td>1,143</td>
<td>1,109</td>
<td>11.7%</td>
<td>5.7%</td>
</tr>
<tr>
<td>Oil seeds and oleaginous fruits</td>
<td>1,125</td>
<td>1,012</td>
<td>1,002</td>
<td>10.6%</td>
<td>8.0%</td>
</tr>
<tr>
<td>Protein crops</td>
<td>29</td>
<td>32</td>
<td>34</td>
<td>0.4%</td>
<td>3.0%</td>
</tr>
<tr>
<td>Raw tobacco</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0.0%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Sugar beet</td>
<td>39</td>
<td>50</td>
<td>32</td>
<td>0.3%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Other industrial crops</td>
<td>42</td>
<td>48</td>
<td>39</td>
<td>0.4%</td>
<td>2.4%</td>
</tr>
<tr>
<td><strong>Fodder plants:</strong></td>
<td>1,705</td>
<td>1,465</td>
<td>1,314</td>
<td>13.9%</td>
<td>5.4%</td>
</tr>
<tr>
<td>Vegetables and horticultural products</td>
<td>2,024</td>
<td>2,021</td>
<td>1,878</td>
<td>19.9%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Potatoes</td>
<td>1,280</td>
<td>1,161</td>
<td>678</td>
<td>7.2%</td>
<td>7.1%</td>
</tr>
<tr>
<td>Fruits</td>
<td>1,087</td>
<td>1,137</td>
<td>1,093</td>
<td>11.6%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Wine</td>
<td>306</td>
<td>240</td>
<td>185</td>
<td>2.0%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Olive oil</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Other crop products</td>
<td>35</td>
<td>19</td>
<td>10</td>
<td>0.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td><strong>Crop output:</strong></td>
<td>12,185</td>
<td>11,040</td>
<td>9,450</td>
<td>78.2%</td>
<td>4.5%</td>
</tr>
<tr>
<td><strong>Animals:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cattle</td>
<td>303</td>
<td>271</td>
<td>333</td>
<td>8.3%</td>
<td>1.0%</td>
</tr>
<tr>
<td>Pigs</td>
<td>965</td>
<td>896</td>
<td>779</td>
<td>15.4%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Equines</td>
<td>22</td>
<td>19</td>
<td>22</td>
<td>0.6%</td>
<td>2.2%</td>
</tr>
<tr>
<td>Sheep and goats</td>
<td>196</td>
<td>253</td>
<td>225</td>
<td>5.5%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Poultry</td>
<td>425</td>
<td>451</td>
<td>441</td>
<td>11.0%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Other animals</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td><strong>Animal products:</strong></td>
<td>1,996</td>
<td>2,076</td>
<td>2,207</td>
<td>55.1%</td>
<td>3.4%</td>
</tr>
<tr>
<td>Milk</td>
<td>1,012</td>
<td>1,106</td>
<td>1,111</td>
<td>27.7%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Eggs</td>
<td>662</td>
<td>685</td>
<td>777</td>
<td>19.4%</td>
<td>2.1%</td>
</tr>
<tr>
<td>Other animal products</td>
<td>322</td>
<td>285</td>
<td>319</td>
<td>8.0%</td>
<td>2.1%</td>
</tr>
<tr>
<td><strong>Animal output:</strong></td>
<td>3,908</td>
<td>3,967</td>
<td>1,801</td>
<td>29.8%</td>
<td>2.5%</td>
</tr>
<tr>
<td><strong>Agricultural goods output:</strong></td>
<td>16,092</td>
<td>15,007</td>
<td>13,458</td>
<td>100.0%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Gross value added at basic prices</td>
<td>7,621</td>
<td>7,096</td>
<td>6,444</td>
<td>4.0%</td>
<td></td>
</tr>
</tbody>
</table>

Source: Eurostat, Economic Accounts for Agriculture (values at current producer prices).

*Updated March 2018*

**Figure:** Agricultural Output per Type, Romania, 2013 (% share of utilised agricultural area)
Farming Data

The Group Roullier provided us a part of their clients database with:

- Precise geolocalisation of each farm
- Accounting information about more than 12,000 farms located in Romania for the last 5 years
- Types of crop and number of hectares cultivated per farm
- Yields per hectare per crop per region since 1990
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We collected the market prices time series for the five main crops:

- Wheat
- Corn
- Barley
- Sunflower
- Rapeseed
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The Group Roullier provided us a part of their clients database with:

- Precise geolocalisation of each farm
- Accounting information about more than 12,000 farms located in Romania for the last 5 years
- Types of crop and number of hectares cultivated per farm
- Yields per hectare per crop per region since 1990

We collected the market prices time series for the five main crops:

- Wheat
- Corn
- Barley
- Sunflower
- Rapeseed

We got access to a European weather database with the following characteristics:

- Daily precipitations (over more than 20 years)
- Daily mean, max and min temperatures (over more than 20 years)
- For 40 different weather stations in Romania, Ukraine, Moldova, Hungary and Serbia
### Data Description

#### Data

**Table:** Crops growing seasons and corresponding critical growing period

<table>
<thead>
<tr>
<th>2 Crops</th>
<th>Whole growing period</th>
<th>critical growing period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat (W)</td>
<td>Sep/Oct - July/Aug</td>
<td>April - July</td>
</tr>
<tr>
<td>Corn (C)</td>
<td>Apr/May - Aug/Sep</td>
<td>June - Aug</td>
</tr>
</tbody>
</table>

(a) Corn arable hectares  
(b) Wheat arable hectares
**Data**

- We compare several models:

<table>
<thead>
<tr>
<th>Group</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Most nearest station(region central point)</td>
</tr>
<tr>
<td></td>
<td>(2) Most nearest station(farm level simple average)</td>
</tr>
<tr>
<td>Without GP</td>
<td>(3) Weighted inverse distance</td>
</tr>
<tr>
<td></td>
<td>(4) Weighted farm hectares</td>
</tr>
<tr>
<td></td>
<td>(5) Weighted distance-hectares</td>
</tr>
<tr>
<td></td>
<td>(6) Simple average</td>
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<tr>
<td>With laGP</td>
<td>(7) Weighted inverse distance</td>
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<td>(8) Weighted farm hectares</td>
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<td></td>
<td>(9) Weighted distance-hectares</td>
</tr>
<tr>
<td></td>
<td>(10) Simple average</td>
</tr>
<tr>
<td>With GPfit</td>
<td>(11) Weighted inverse distance</td>
</tr>
<tr>
<td></td>
<td>(12) Weighted farm hectares</td>
</tr>
<tr>
<td></td>
<td>(13) Weighted distance-hectares</td>
</tr>
</tbody>
</table>

- **Physical distance** $\varphi$ between any two locations given longitude $\lambda_x$ and latitude $\omega_x$ is measured as in (Norton et al., 2012):

$$
\varphi = R \cdot \cos^{-1} (\sin(\omega_1) \cdot \sin(\omega_2) + \cos(\omega_1) \cdot \cos(\omega_2) \cdot \cos(\lambda_2 - \lambda_1))
$$

where R is a constant stand for the radius of the sphere (3963.1 miles).
## Results

### Model Fitting Quality

<table>
<thead>
<tr>
<th>Region</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\Delta A^2$</th>
<th>LB-Q</th>
<th>Arch</th>
<th>KS</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Cossana</td>
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<tr>
<td>Dolj</td>
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<td>11.47**</td>
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<td>0</td>
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<tr>
<td>Gurbajal</td>
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<td>94.77**</td>
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</tbody>
</table>

### Data and Results

- **Model Fitting Quality**
- **Region**
- **$\alpha_1$**
- **$\alpha_2$**
- **$\alpha_3$**
- **$\Delta A^2$**
- **LB-Q**
- **Arch**
- **KS**
- The SARIMA time series coupled spatial Gaussian process model exhibits distinguishable superiority compared with nonGP approach.

<table>
<thead>
<tr>
<th>Confidence interval</th>
<th>Group</th>
<th>Model (5)</th>
<th>Model (9)</th>
<th>Model (13)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Wheat</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Whole regions</td>
<td>41%</td>
<td>44%</td>
<td>68%</td>
</tr>
<tr>
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<td>Prolific zone</td>
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<td>75%</td>
<td>71%</td>
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<tr>
<td></td>
<td>Wheat</td>
<td>73%</td>
<td>75%</td>
<td>93%</td>
</tr>
<tr>
<td></td>
<td>Whole regions</td>
<td>71%</td>
<td>75%</td>
<td>93%</td>
</tr>
<tr>
<td></td>
<td>Prolific zone</td>
<td>76%</td>
<td>75%</td>
<td>95%</td>
</tr>
<tr>
<td></td>
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<td>76%</td>
<td>75%</td>
<td>98%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>68%</td>
<td>88%</td>
<td>98%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>71%</td>
<td>100%</td>
<td>98%</td>
</tr>
</tbody>
</table>

- Weighted distance-hectares ratio method (Model (5), Model (9) and Model (13)) provides us the best estimation results compared with the other weighting methods.
Results

Farms Size Dispersion

- degree of dispersion of the farms crop size characterizing the region of Constanta, a highly productive area of the south east of Romania

(c) Corn arable hectares

(d) Wheat arable hectares
### Results

#### The Balance Sheet

Credit risk profile through Weather risk model at 0.95 quantile weather condition

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1987 sample</th>
<th></th>
<th></th>
<th>103 sample</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Correl=0</td>
<td>Correl=1</td>
<td>Benchmark</td>
<td>Correl=0</td>
<td>Correl=1</td>
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<tr>
<td>Actual farm obs.</td>
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<td>1442</td>
<td>1442</td>
<td>63</td>
<td>63</td>
<td>63</td>
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<tr>
<td>Mean Prob. of default</td>
<td><strong>5.79%</strong></td>
<td><strong>5.79%</strong></td>
<td><strong>5.79%</strong></td>
<td><strong>6.20%</strong></td>
<td><strong>6.20%</strong></td>
<td><strong>6.20%</strong></td>
</tr>
</tbody>
</table>

Credit risk profile through Weather risk model under mean weather condition

<table>
<thead>
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<th>Mean Prob. of default</th>
<th></th>
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<th></th>
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<tbody>
<tr>
<td></td>
<td><strong>4.69%</strong></td>
<td><strong>4.69%</strong></td>
<td><strong>4.69%</strong></td>
<td><strong>3.18%</strong></td>
<td><strong>3.18%</strong></td>
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</tbody>
</table>

Credit risk profile through commodity default risk model

<table>
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<th>Statistic</th>
<th>1987 sample</th>
<th></th>
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<tr>
<td>Actual farm obs.</td>
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<td>1442</td>
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<tr>
<td>Mean Prob. of default</td>
<td><strong>4.66%</strong></td>
<td><strong>4.66%</strong></td>
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<td><strong>3.98%</strong></td>
<td><strong>3.98%</strong></td>
<td><strong>3.98%</strong></td>
</tr>
</tbody>
</table>

**More diversified than average for standard weather conditions**

**Less diversified than average for extreme weather conditions**
Introduction

Credit Risk
- Accounting basics
- Merton’s Model
- Farm’s Asset Modelling

Commodity Risk

Weather Risk
- Review of the Literature
- Time and Space Decomposition
- Two Types of Gaussian Process
- The Yield Model

Data and Results
- Data Description
- Models Comparison
- Results

Conclusion
We proposed a credit risk model taking into account the impacts of the weather conditions upon farmers profits.
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We combine GP with a SARIMA time-series model for handling spatio-temporal weather information.
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We combine GP with a SARIMA time-series model for handling spatio-temporal weather information.

Through our model we distinguish a global and a local risk of credit dependence.
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We combine GP with a SARIMA time-series model for handling spatio-temporal weather information.

Through our model we distinguish a global and a local risk of credit dependence.

We treat separately the conditional loss distribution for the commodity risk and the conditional loss distribution associated to the weather risk.
We proposed a credit risk model taking into account the impacts of the weather conditions upon farmers profits.

We combine GP with a SARIMA time-series model for handling spatio-temporal weather information.

Through our model we distinguish a global and a local risk of credit dependence.

We treat separately the conditional loss distribution for the commodity risk and the conditional loss distribution associated to the weather risk.

If we assume the two sources of risk as independent we also proposed a version where both risk sources are included.

... Thank you!
Thank you